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Paramagnetic limit in ferromagnetic superconductors with triplet pairing

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The spin susceptibility in the ferromagnetic superconductors with triplet pairing is calculated. It is shown that the absence of the superconductivity paramagnetic limitation for field directions perpendicular to the direction of the spontaneous magnetization is explained by the itinerant ferromagnet band splitting rather than by an adjustment of equal spin-pairing state to magnetization direction.

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I. INTRODUCTION

It is commonly believed that in the ferromagnet superconducting uranium compounds,[1–3] we deal with triplet superconductivity. In particular, this is due to the fact that the upper critical field strongly exceeds the Pauli limit. However, the paramagnetic limitation of triplet superconductivity is unimportant only when the external field is directed parallel to the spin-quantization axis but is quite essential when the field is oriented perpendicular to the spin-quantization direction.[4,5] In reality, the situation is the opposite. In two of the uranium superconducting compounds URhGe and UCoGe, the upper critical field in the direction of spontaneous magnetization $\hat{c}$ is approximately the paramagnetic limiting field in these materials. At the same time, the upper critical field in the directions perpendicular to the magnetization is much higher than the paramagnetic limiting field.[6–8] Moreover, this property persists also in the reentrant superconducting state of the URhGe,[9,10] where superconductivity reappears under a magnetic field of 12 T in the b crystallography direction causing alignment of the magnetization parallel to the b axis. The additional field oriented in the a-crystallography direction does not destroy the superconducting state until 20 T. Similar behavior was recently found in UCoGe.[11,12]

In the superfluid ^{3}He-A, pairing occurs between quasiparticles with equal spin projections (up-up or down-down) on the direction of spin-quantization axis. The latter adjusts itself in the direction of the external field. This case the susceptibility of A phase keeps its normal-state value that prevents the paramagnetic suppression of superfluid state.[4] The same should be in a superconducting state with equal spin pairing in the absence of spin-orbital coupling fixing the mutual orientation of the spin-quantization axis and the crystalline symmetry direction. In uranium compounds, the magnetic anisotropy is quite strong.[13] In the absence of an external field here we deal with equal spin-pairing state with spin-quantization axis parallel to the direction of spontaneous magnetization.[14,15] The zero-temperature upper critical field parallel to the b axis in URhGe is approximately 1.3 T.[6] This field causes only a tiny rotation of the magnetization direction[9] that means the pairing still occurs in equal spin-pairing state with the same (as in the zero field) direction of quantization axis perpendicular to the external field. At the same time, this field exceeds more than twice the paramagnetic limiting field[6] but does not destroy the superconducting state.

Here we investigate theoretically such a remarkable behavior of the ferromagnetic superconductors. We shall give the microscopic derivation of the paramagnetic susceptibility of the ferromagnetic superconductors for the field orientation perpendicular to the direction of the spontaneous magnetization. The absence of Pauli limitations of superconductivity is found to be related to the itinerant ferromagnet band splitting. The adjustment of the equal spin-pairing state to the external field direction is also important at higher fields near the metamagnetic or magnetization rotation phase transition. Hence, the critical field in the itinerant ferromagnets can be calculated ignoring the paramagnet limitations. In conclusion, we discuss the upper critical-field temperature dependence in the uranium compounds in the moderate field region.

II. SPIN SUSCEPTIBILITY

URhGe and UCoGe are the orthorhombic ferromagnets with spontaneous magnetization oriented along the c-crystallography axis. At temperatures below the Curie temperature and in the absence of a magnetic field, the c component of magnetization has a finite value. The magnetic field H applied along the b axis creates a magnetization along its direction but decreases the magnetization parallel to c. We shall denote by y,z the directions of the spin axes pinned correspondingly to the (b,c) crystallographic axis. The field dependence of magnetization components in URhGe has been reported in the Ref. 9. For a superconducting state realized in the low-field region of the phase diagram, the upper critical field for the field orientation along the b axis does not exceed 1.3 T.[6] At this field, the magnetization in the b direction is at least ten times smaller than the magnetization along the c direction, which is practically field independent.[9] Hence, the magnetic field acting on the electron spins in the $\hat{z}$ direction can be taken equal to the exchange field,

$$\mathbf{h} = h\hat{z}. \tag{1}$$

The field acting on the electron spins in the $\hat{y}$ direction is given by the external field,[16]

$$\mathbf{H} = H\hat{y}. \tag{2}$$

In what follows we shall assume that both the phenomena of ferromagnetism and superconductivity are determined by the spin-up and the spin-down electrons filling two separate bands split by the exchange field $h \sim T_c/\mu_B$. Then the magnetic moment of the itinerant electron subsystem is given by

$$\mathbf{M} = \mu_B T \sum_n \int \frac{d^3k}{(2\pi\hbar)^3} \mathrm{Tr}\, \boldsymbol{\sigma} \hat{G}. \tag{3}$$

Here $\boldsymbol{\sigma}=(\sigma_x,\sigma_y,\sigma_z)$ are the Pauli matrices.

In the normal state, the Green's function in the approximation of being linear in H is

$$\hat{G} = \hat{G}_n - \mu_B H \hat{G}_n \sigma_y \hat{G}_n, \tag{4}$$

where

$$\hat{G}_n = \begin{pmatrix} G_{n+} & 0 \\ 0 & G_{n-} \end{pmatrix}, \quad G_{n\pm} = \frac{1}{i\omega_n - \xi_{\mathbf{k}} \pm \mu_B h}. \tag{5}$$

We obtain

$$\mathbf{M}_n = \mu_B T \sum_n \int \frac{d^3k}{(2\pi\hbar)^3} [\hat{z}(G_{n+} - G_{n-}) - 2\mu_B H \hat{y} G_{n+} G_{n-}], \tag{6}$$

which can be rewritten as

$$\mathbf{M}_n = \mu_B T \sum_n \int \frac{d^3k}{(2\pi\hbar)^3} (\hat{z} + H\hat{y}/h)(G_{n+} - G_{n-}). \tag{7}$$

For a finite value of the exchange field, this is equal to

$$\mathbf{M}_n = \mu_B \frac{N_\uparrow - N_\downarrow}{h} (\mathbf{h} + \mathbf{H}). \tag{8}$$

Here $N_{\uparrow,\downarrow}$ are the numbers of electrons in the spin-up and spin-down band. The corresponding susceptibility is

$$\chi_{yy}^n = \mu_B \frac{N_\uparrow - N_\downarrow}{h}. \tag{9}$$

On the other hand, in the absence of the band splitting, that is, at $h=0$, the magnetic moment is

$$\mathbf{M}_n(h=0) = 2\mu_B^2 N_0 \mathbf{H}, \tag{10}$$

where N_0 is the density of states per one-electron spin projection. The susceptibility is given by the Pauli formula

$$\chi_{yy}^n(h=0) = 2\mu_B^2 N_0. \tag{11}$$

The superconducting state in the two-band itinerant ferromagnet is built of pairing states formed either by spin-up electrons from one band or by spin-down electrons from another band.[14,15] This state is characterized by a two-component order parameter,

$$\hat{\Delta} = \begin{pmatrix} \Delta_{\mathbf{k}\uparrow} & 0 \\ 0 & \Delta_{\mathbf{k}\downarrow} \end{pmatrix}. \tag{12}$$

Then instead of Eq. (6) we obtain

$$\mathbf{M}_s = \mu_B T \sum_n \int \frac{d^3k}{(2\pi\hbar)^3} [\hat{z}(G_{s+} - G_{s-}) - \mu_B H \hat{y}(G_{s+}G_{s-} + G_{s-}G_{s+} + F_+F_-^\dagger + F_-F_+^\dagger)], \tag{13}$$

where

$$G_{s\pm} = \frac{-i\omega_n - \xi_{\mathbf{k}\pm}}{\omega_n^2 + \xi_{\mathbf{k}\pm}^2 + |\Delta_{\uparrow,\downarrow}|^2}, \quad F_\pm = \frac{\Delta_{\uparrow,\downarrow}}{\omega_n^2 + \xi_{\mathbf{k}\pm}^2 + |\Delta_{\uparrow,\downarrow}|^2} \tag{14}$$

are the superconducting-state Green's functions and

$$\xi_{\mathbf{k}\pm} = \xi_{\mathbf{k}} \mp \mu_B h.$$

The magnetic moment induced by the external field is determined by the second line in Eq. (13). This line contains the products of the Green's functions from the different bands. As in the normal state given by Eqs. (6) and (7) the superconducting-state magnetic moment is nothing but the response of the magnetic moments of the electrons filling the momentum-space shell "sandwiched" between the Fermi surfaces of the spin-up and spin-down bands.[17] The phase transition to the superconducting state changes the Fermi distribution of the electrons only over energies close to the corresponding Fermi surfaces within a range, of order $\Delta_{\uparrow,\downarrow}$. Hence, the integration of the second line in Eq. (13) leads to a result only slightly different from its normal-state value. More exactly, a straightforward calculation shows that at a band splitting exceeding the superconducting gaps $\mu_B h \gg |\Delta_\pm|$, even at $T=0$,

$$\frac{T\sum_n \int d\xi [2G_{n+}G_{n-} - 2G_{s+}G_{s-} - F_+F_-^\dagger - F_-F_+^\dagger]}{T\sum_n \int d\xi G_{n+}G_{n-}} \sim \sum_{\alpha\beta=\uparrow,\downarrow} \frac{\Delta_{\mathbf{k}\alpha}\Delta_{\mathbf{k}\beta}^*}{(\mu_B h)^2} \ln \frac{(\mu_B h)^2}{\Delta_{\mathbf{k}\alpha}\Delta_{\mathbf{k}\beta}} \ll 1. \tag{15}$$

This implies that the susceptibility in the superconducting state practically keeps its normal-state value,

$$\chi_{yy}^s(T=0) = \chi_{yy}^n \left(1 - a \frac{\Delta^2}{(\mu_B h)^2} \ln \frac{\mu_B h}{|\Delta|}\right). \tag{16}$$

Here, $|\Delta|$ is the characteristic quantity of the gap amplitudes and a is a numerical constant. Hence, the energy of superconducting ferromagnet in a magnetic field at $T=0$ is higher than that of a normal ferromagnet state by an amount of order

$$N_0 \Delta^2 \frac{H^2}{h^2} \ln \frac{\mu_B h}{|\Delta|} \tag{17}$$

while the zero-temperature superconducting-state energy gain is on the order of $N_0\Delta^2$. The comparison of these two energies leads to the conclusion that the paramagnetic limiting field proves to be of the order of the exchange field,

$$H_p \approx \frac{h}{\ln(\mu_B h/|\Delta|)}. \quad (18)$$

Hence, as long as the band splitting is larger than the gap, the paramagnetic suppression of the superconducting state by the field perpendicular to the spontaneous magnetization is absent.

On the contrary, at $h=0$ a formal calculation from Eq. (13) yields the susceptibility,

$$\chi_{yy}^s(h=0,T) = 2\mu_B^2 N_0 \int \frac{d\Omega}{4\pi} Y(\hat{\mathbf{k}},T), \quad (19)$$

where

$$Y(\hat{\mathbf{k}},T) = \frac{1}{4T}\int_{-\infty}^{+\infty} \frac{d\xi}{\cos\text{h}^2(\sqrt{\xi^2+\Delta_{\mathbf{k}}^2}/2T)}$$

is a generalized Yosida function. The susceptibility $\chi_{yy}(h=0,T)$ tends to zero as $T\rightarrow 0$.[4] Thus, the magnetic field directed perpendicular to the spins of the Cooper pairs in a nonferromagnetic superconductor with triplet pairing suppresses superconductivity as it does in the usual superconductors with singlet pairing.

III. CONCLUDING REMARKS

In framework of the weak-coupling theory of superconductivity, there are three mechanisms of the magnetic field influence on the superconducting state in the superconductors with triplet pairing:[18] (i) orbital depairing, (ii) paramagnetic limiting, and (iii) stimulation or suppression of nonunitary superconductivity due to magnetic field dependence of density of states.[19]

We have demonstrated that the superconducting state in the itinerant ferromagnet superconductors with triplet pairing is not subject to paramagnetic limiting. Here we describe the situation with two other mechanisms of superconductivity suppression, in case of moderate fields directed perpendicular to spontaneous magnetization.

The superconducting-state spontaneous magnetization is determined by the first line of Eq. (13). Calculation taking into account the asymmetry of the particle-hole distributions near the Fermi surfaces yields[15]

$$\mathbf{M}_s^{sp} = \hat{z}\mu_B\left[N_\uparrow - N_\downarrow + (N'_{0\uparrow}|\Delta_\uparrow|^2 - N'_{0\downarrow}|\Delta_\downarrow|^2)\ln\frac{\varepsilon_F}{T_s}\right]. \quad (20)$$

The spontaneous magnetization is slightly modified in comparison with its normal-state value,

$$\mathbf{M}_n^{sp} = \hat{z}\mu_B(N_\uparrow - N_\downarrow). \quad (21)$$

Here, $N'_{0\uparrow}$ and $N'_{0\downarrow}$ are the energy derivatives of the density of states at the Fermi level for the spin-up and spin-down band, respectively. The change in spontaneous magnetization causes a corresponding energy shift at the phase transition to the superconducting state that is equal to $-(\mathbf{M}_s^{sp}-\mathbf{M}_n^{sp})\mathbf{h}$. This leads in turn to the superconducting critical temperature shift proportional to the exchange field. To avoid presenting the cumbersome formula we write it for the case of where only the spin-up pairing is present,

$$\frac{\delta T_s}{T_s} = \mu_B h \frac{N'_{0\uparrow}}{N_{0\uparrow}}\ln\frac{\varepsilon_F}{T_s} \approx \pm\frac{\mu_B h}{\varepsilon_F}\ln\frac{\varepsilon_F}{T_s}. \quad (22)$$

This shift takes place independently of whether the pairing mechanism has a magnetic or nonmagnetic origin.

A magnetic field $\mathbf{H}$ directed perpendicular to the direction of spontaneous magnetization does not cause a shift linear in H in the free energy of the superconducting state. As we have already mentioned, the magnetization along the $\hat{z}$ direction is practically field independent. Hence, for this field direction the third mechanism of magnetic field influence on the superconducting state is also ineffective. This statement is valid in moderate fields as long as $M_z(H)\approx M_z(0)$.

Thus, only the orbital mechanism of suppression of superconductivity is essential. Experimentally, in UCoGe for the field directed perpendicular to the spontaneous magnetization, it was observed a pronounced upper critical-field upward curvature.[7,8,11] A possible explanation for this behavior is related to the crossover between two phases in a two-band ferromagnetic superconductor.[14,15]

In conclusion, we have demonstrated that the absence of paramagnetic limitations of superconductivity in ferromagnetic superconductors for the field direction perpendicular to the spontaneous magnetization is related to the itinerant ferromagnet band splitting rather than to an adjustment of equal spin-pairing state to the external field direction. Thus, the upper critical field can noticeably exceed the usual paramagnetic limiting field value $H_p \approx T_{sc}/\mu_B$. The conclusions formulated above are valid at moderate magnetic fields when $M_z(H)\approx M_z(0)$. At external fields on the order of the exchange field $H\sim h$, the equilibrium magnetization aligns itself parallel to the external field. In these conditions, the paramagnetic limitation of superconductivity in itinerant ferromagnets is absent as well.

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